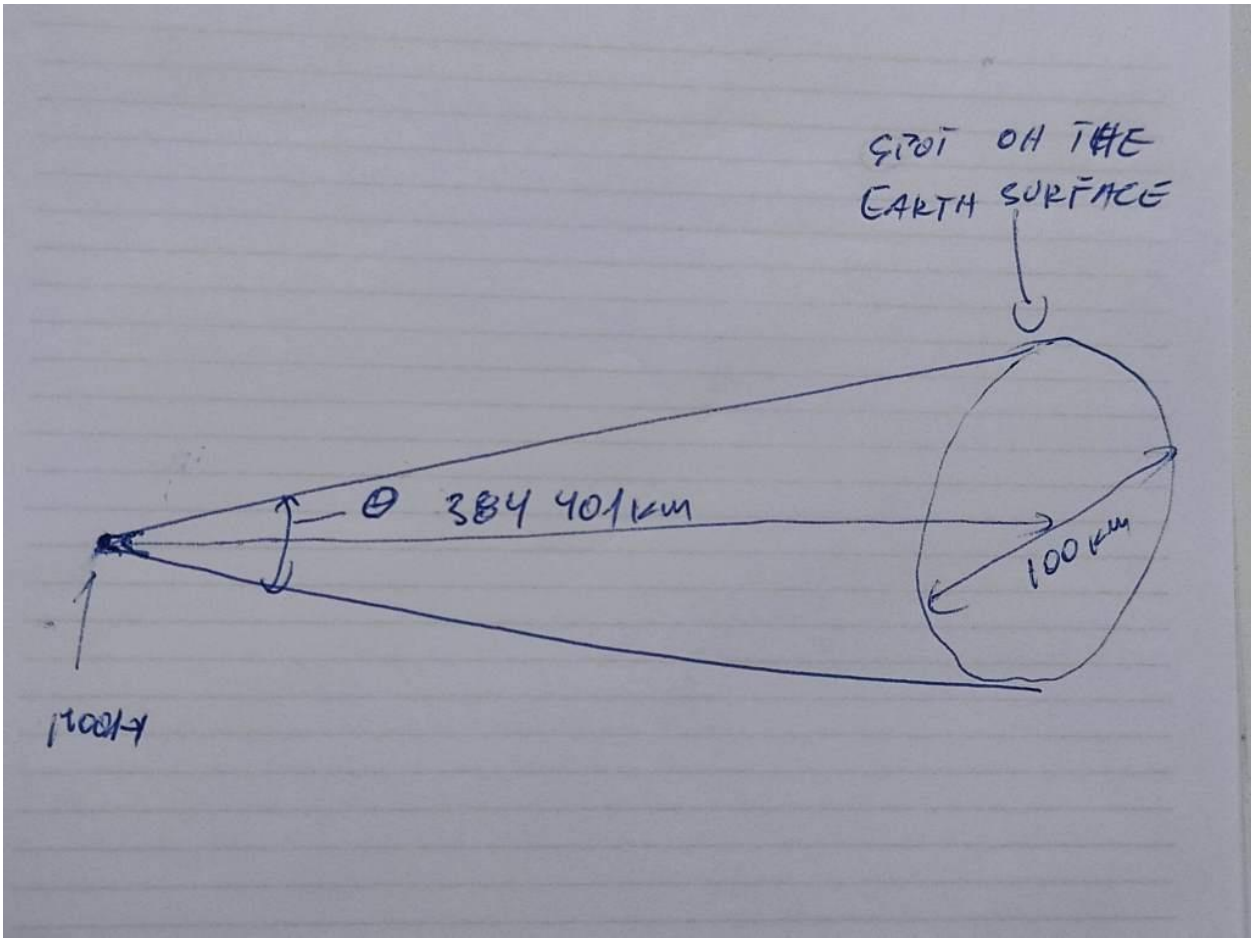
General considerations

Moon to Earth distance (avg) 384 401 km

Spot diameter on the Earth surface 100km



Divergence angle

Θ = 384401km/100km= $2.6 \cdot 10^{-4}$ rad = 0.015 deg

1. Estimation of the laser power needed according to the human eye sensitivity (the lowest needed power)

Assumptions and definitions:

1.1. According to the recent studies a human eye can see the candle light at distance cca l=2.6km under realistic conditions (background light of the night sky etc) [1,2,3]. The distance of 48km stated in psychology textbooks seems to be too optimistic [4]

1.2. A common wax candle emits light with a luminous intensity of roughly one candela 1cd [5 pg 227, 6].

1.3. An average diameter of a human eye's pupil is approximately d=8mm assuming the night conditions when the pupil is maximally dilated [7,8,9]

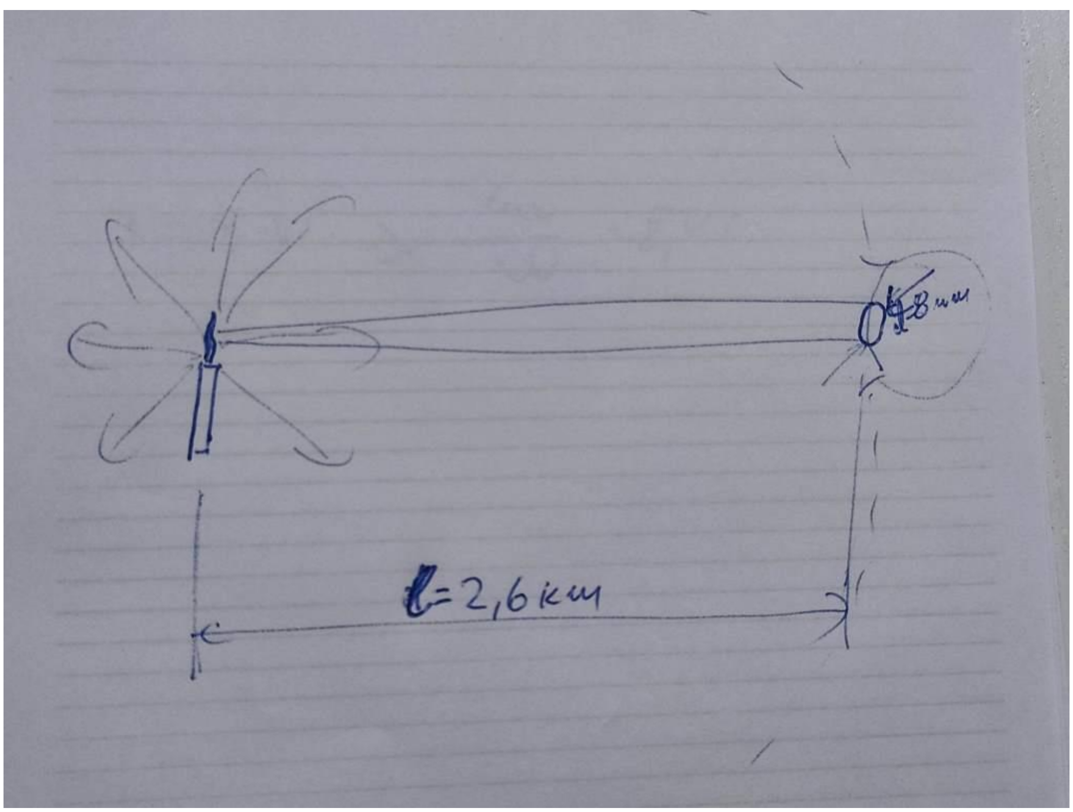
1.4. The unit for luminous flux or luminous power is lumen = candela steradians; 1lm = 1cd · 1sr

1.5. 1W optical power has 683lm at 555nm wavelength, assuming scotopic vision [10,11]

1.6. Tabulated values of luminosity function are given in [12]

According to 1.2 and 1.4 one candle has luminosity power of $4\pi \approx 12.56$ lm (full solid angle is 4π sr)

Taking into account 1.1 and 1.3 one can calculate the luminous power Φ_{ep} at the eye's pupil at the distance l=2.6km from the candle given the area around the candle is $A_{sphere}=4\pi l^2$ and area of the eye's pupil is $A_{ep}=\pi d^2/4$



Φ_{ep}= 12.56 lm · (A_{ep} / A_{sphere}) = 7.43·10^{-12}lm

Whilst the above result is not necessary in the present calculation (the other way around is to use the illuminance, see the next section) it might be useful and interesting since it express the minimal luminous power that human eye is still able to observe in scotoptic vision.

According to 1.5 and 1.6 the corresponding optical power at the eye's pupil is P=7.43·10^{-12}lm/683lm/W= 1.09·10^{-14}W. The latter value stands for 555nm wavelength. For another wavelength, e.g. 532nm which is the typical frequency doubled Nd YAG laser (e.g. pointer), one has to divide the obtained power by 0.89.

Finally, one comes to optical power at the detection threshold of an eye at 532nm wavelength

P_{eyeth} = 1.22·10^{-14}W = 122fW

The minimal CW (continuous wave) laser power needed to achieve human eye sensitivity at the 100km diameter area on the Earth surface (the first skatch) would be:

P_{las} = 122fW $(100km)^2/(8mm)^2$= 1.91 W

Nota bene: the scattering and the absorption through the Earth's atmosphere would increase the minimal laser power further more. A very rough estimation is that 80% of incoming sunlight reaches the Earth's surface [13]. Thus, the above laser power should be corrected to the

P_{las} = (10/8)·1.91W = 2.39W

Using a pulsed laser above value can be regarded as a peak power, thus yielding the following pulse energies E_p for various pulse durations Δt_p:

Δt_p = 1ms => E_p=2.39mJ

Δt_p = 1µs => E_p=2.39µJ

Δt_p = 1ns => E_p=2.39nJ

Here, one has to take into account the minimal light pulse duration detectable by a human eye

2. Estimation of the laser power needed according to the moonlight intensity on the Earth's surface

The laser power need to be seen from the earth can be estimated according to the moonlight intensity. In other words, if a human eye can see the moonlight, than it will see the laser light of the approximately same intensity.

Assumptions and definitions:

2.1. Illuminance is defined as luminous power (in lumens) incident on a surface (in square meters). The unit for illuminance is lux, thus $1lx=1lm/1m^2$ [10]

2.2. Illuminance of the moonlight on the Earth's surface during the full moon is 0.05-0.1lx [14] taking into account the Earth atmosphere influence.

According to the lowest illuminance of the full moon from 2.2, one can calculate the illuminous power needed to illuminate the area of 100km in diameter on the Earth's surface

$\Phi_{100km} = 0.05lx \cdot (100km/2)^2\pi = 392\ 699\ 081.698lm$

According to 1.5 and 1.6 the corresponding total optical power of the moonlight at 100km diameter area is P=392 699 081.698lm/683lm/W= 574.962kW. The latter value stands for 555nm wavelength which is rough approximation for the moonlight.

If we assume that a human eye can see easily the light form the 1/10 of the moon, finally we get the CW

$P_{las} = 57.4962kW$

Nota bene: In this case it again necessary to make correction to the obtained laser power due tothe scattering and the absorption through the Earth's atmosphere. A very rough estimation is that 80% of incoming sunlight reaches the Earth's surface [13]. Thus, the above laser power corrected for the atmosphere influence is:

$$P_{las} = (10/8)\cdot 57.4962 = 71.87\ kW$$

Using a pulsed laser above value can be regarded as a peak power, thus yielding the following pulse energies E_p for various pulse durations Δt_p:

$\Delta t_p = 1ms \Rightarrow E_p=71.87J$

$\Delta t_p = 1\mu s \Rightarrow E_p=71.87mJ$

$\Delta t_p = 1ns \Rightarrow E_p=71.87\mu J$

Here, one has to take into account the minimal light pulse duration detectable

3. Estimation of the laser power needed according to the aviation obstacle warning lights.

The laser power need to be seen from the earth can be estimated according to the properties of the warning lights in aviation. In other words, if the light of certain luminosity can be seen within certain angle by a pilot's eye, than the laser light of the approximately same intensity at the Earth surface will be observable by an observer eye.

Assumptions and definitions:

3.1 According to the data sheet (of a randomly chosen manufacturer) the obstruction green light of 50cd is visible from a distance of R = 10km at an angle $2\alpha = 10^{o}$ [15]

DL50S & DL50D Low Intensity Aviation Obstruction Light

Description

LANSING DL50 series is typically a standard LED Obstruction light designed for marking any obstacle that may present hazards to aircraft navigation. It made of die-cast aluminum alloy housing with polycarbonate dome and complies with ICAO regulation, low intensity TYPE C.

Features& Benefits:

- Based on Led Technologies
- Customizable for both AC and DC conditions.
- Customizable for steady burning or flashing mode(Flashing rate adjustable)
- The visible distance is more than 10KM
- Multiple light elements in the source reduce the chances of a complete blackout which could be caused in case of a single source failure.
- All hardware of obstruction lights are made of corrosion resistant metals.
- Bird spikes for protection against droppings.
- Built in photocell, which can switch ON/OFF the lights automatically at dawn and dusk (optional)
- Use of quality components and sophisticated technology reduce chances of maintenance and replacement.
- Low power consumption, which leads to huge savings on electricity bills.
- LED light source ensures long lifetime, maintenance free
- Built-in anti electroplate impact module, with lightning/surge protection
- Resistant to shock and vibration.
- No RF Radiations, EMC Compliant
- Reversed Polarity Protected(DC Type)
- Corrosion resistant lamp & housing-suitable for off-shore use
- Bottom cable entry-ease of mount, mount accessories available
- With dry contact alarm, it will give an alarm in case of power and light failure
- Available both in single and double type
- With GPS function, can work with other lights synchronously(optional).

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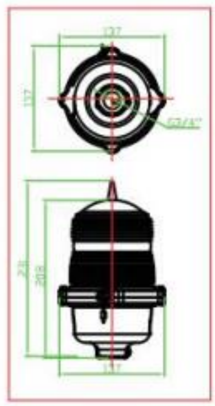
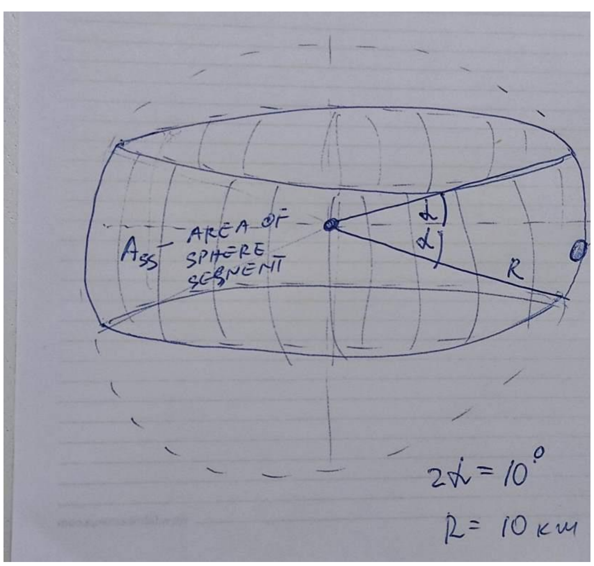
DL50S DL50D

Specifications

LIGHT OUT-PUT	DL50S(Single)	DL50DST(Double)	DL50DSS(Double)
Effective Intensity	50 Candela	50 Candela	50 Candela
Vertical Divergence	10 Degree		
Horizontal out-put	360 Degree		
LED Color	Yellow/blue/red green		
LED Lifespan	100,000 Hours		
OPERATION			
Flash Pattern	Steady burning/Flashing(flashing rate can be customized)		
POWER SUPPLY			
Input voltage	110-220VAC, 50/60 Hz;12VDC;24VDC;48VDC;220VDC		
Frequency	50Hz ~ 60Hz		
MECHANICAL STRUCTURE			



According to 1.2 and 1.4 the warning light has luminosity power of $4\pi \cdot 50\text{cd} \approx 628\ \text{lm}$



Given the area of the sphere segment is A_{ss}=4πR^2sinα

one obtains the illuminance on the sphere segment 628lm/A_{ss}=5.74·10^{-6}lx

Then, the illuminous power needed to illuminate the area of 100km in diameter on the Earth's surface

Φ_{100km} = 5.74·10^{-6}lx · $(100km/2)^2\pi$ = 45 024.324lm

According to 1.5 and 1.6 the corresponding total optical power of the warning light at 100km diameter area is P = 45 024.324lm/683lm/W= 65.94W. The latter value stands for 555nm wavelength which is rough approximation for the green light of the warning light.

For another wavelength, e.g. 532nm which is the typical frequency doubled Nd YAG laser (e.g. pointer), one has to divide the obtained power by 0.89. Finally we get the CW

P_{las} = 74.09W

Nota bene: the scattering and the absorption through the Earth's atmosphere would increase the minimal laser power further more. A very rough estimation is that 80% of incoming sunlight reaches the Earth's surface [13]. Thus, the above laser power should be corrected to the

P_{las} = (10/8)·74.09W = 92.61 W

Using a pulsed laser above value can be regarded as a peak power, thus yielding the following pulse energies E_p for various pulse durations Δt_p:

Δt_p = 1ms => E_p=92.61mJ

Δt_p = 1µs => E_p=92.61µJ

Δt_p = 1ns => E_p=92.61nJ

Here, one has to take into account the minimal light pulse duration detectable by a human eye

References:

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